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Final Technical Report

Grant No.: AFOSR-89-0384

Research title: On two problems in experimental design and estimation"

PI: Dr. Dan Voss, Associate Professor of Statistics

Date: July 19, 1993

This report represents the culmination of four years of research supported by AFOSR. The ultimate objective has been to identify a largest class of regular, proper, single-replicate factorial blocks designs. The more focused objective has been to establish a conjecture in experimental design concerning the comparison of two specific classes of such designs. Designs in the first class, referred to here as *generalized cyclic designs*, are designs constructable by the generalized cyclic method of Dean and John (1975) and John and Dean (1975) without the use of pseudofactors. Designs in the second class, referred to here as *pseudofactor designs*, are those designs constructable by the generalized cyclic method when each factor is replaced by pseudofactors each with a prime number of levels. Two such designs are said to be *degrees-of-freedom equivalent* if for each main effect or interaction both designs confound the same number of degrees of freedom. The conjecture is that for each generalized cyclic design there exists a degrees-of-freedom equivalent pseudofactor design. If the conjecture is true, then

for the construction of single replicate designs one would only need apply the methods using pseudofactors in the effort to identify a best design for a given experiment.

Let the *order* of a factor be the number of levels associated with it in an experiment. Prior to this research effort, the only results known were these. First, the two classes of designs are degrees-of-freedom equivalent if no factor has order divisible by a prime power (Voss and Dean, 1987). Second, any generalized cyclic design for which the principle block is cyclic (i.e. a cyclic subgroup of the treatment group) has a degrees-of-freedom equivalent generalized cyclic design (Voss, 1988).

Substantial progress has been made over the four year period, with the conjecture having been proven in various special cases. However, a general proof of the conjecture has not been obtained, and progress during the last two reporting periods has been disappointing. Hence, no further support of this research has been requested. Progress made during the research period includes establishment of the following results.

The strongest result known to date is that the conjecture is true if the orders of at most three factors are divisible by a common prime (Voss, 1993a).

That the use of pseudofactors of prime order is sometimes beneficial has also been shown. Specifically, for experiments in which three or more factors have orders divisible by a common

prime, p say, with at least one of the orders divisible by p^2, there exist pseudofactor designs for which there is no degrees-of-freedom equivalent generalized cyclic design (Voss, 1993a).

The following result is somewhat surprising. If for each prime p the orders of at most two factors are divisible by p, then the two classes of designs are degrees-of-freedom equivalent (Voss, 1993a).

A promising direction of further study continues to be with respect to the class of regular designs, since generalized cyclic designs and pseudofactor designs are both regular (Voss, 1993b). However, no progress has been made in this area, and to date methods of design construction based entirely on the property of regularity are limited to designs involving only two factors (Mukerjee and Dean, 1993), a case for which the conjecture is already established (Voss, 1993a).

A minor result has been established which makes the conjecture more interesting, since it eliminates as inadmissible the class of symmetric designs obtained by the classical methods based on Galois fields of prime power order. Specifically, if the order of each factor is the same prime power, say p^k, then for each classical design which can be constructed over GF(p^k), there exists an equivalent pseudofactor design (Voss, 1993c). The proof is by construction utilizing basic properties of

fields. The corresponding manuscript (Voss, 1993c) has not been published.

An avenue that once appeared and my still be quite promising has still not been fruitful. In the first year or two of this research, I visited Giovanni Zacher of the University of Padova, Italy, while he was in Oxford, Ohio. He recommended as relevant a paper by Barnes (1961) on "Lattice Embeddings of Prime Power Groups." Consider a finite Abelian p-group, G, and the corresponding lattice obtained by consideration of the subgroup relation. Furthermore, assume that G has no cyclic subgroup of order larger than p^2. Let H denote the elementary Abelian p-group with $|H|=|G|$. Barnes claimed that the lattice of G could be embedded in the lattice of H, namely, that there exists a one-to-one mapping from the set of subgroups of G into the set of subgroups of H such that the mapping preserves subgroup order and lattice structure. The proof given is constructive in nature and, with careful choice of generators of G and of H, could apparently be adapted and extended to show the following: If no factor has order divisible by the cube of a prime, then for any generalized cyclic design, there exists of a degrees of freedom equivalent pseudofactor design. Unfortunately, there is a hole in the proof given by Barnes. I have not been able to repair the proof, nor have I gotten any report of progress on this matter from Professor Zacher, who indicated more than a year ago that he and a colleague would look into it.

In conclusion, substantial progress has been made on the basic research question. Reviews of the related manuscripts submitted for publication suggest that the problem now falls under the realm of mathematical rather than statistical interest, since the conjecture of interest, while still not proven in general, is believed and assumed true by statisticians in practice.

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